# INTERPENETRATION OF TWO IONIZED GAS CLOUDS-II

V. B. BHATIA

DEPARTMENT OF PHYSICS, S. D. COLLEGE, NEW DELHI-5

AND

J. N. TANDON

DEPARTMENT OF PHYSICS, UNIVERSITY OF DELHI, DELHI-6

(*Received January* 4, 1962)

**ABSTRACT.** Counterstreaming of two ionized gas clouds has been considered by taking account of the gas pressures of the two clouds. It is shown that the counterstreaming will, in general, be unstable

## INTRODUCTION

Counterstreaming of two ionized gas clouds has been the subject of investigation in recent years (Kahn 1957, Parker 1958, Tandon 1961). It was shown that the counterstreaming, in general, is unstable except when the density of one of the streams is extremely low as compared with that of the other. Kahn suggested that the counterstreaming will be stopped because of this instability and a shock will be generated. Alternatively, Tandon (hereafter referred to as paper I) is of the opinion that the counterstreaming will not stop but the double stream will break into small clouds of space charge with a maximum length given by

$$\lambda_{max.} = \sqrt{\frac{\pi m M U^2}{2N_0 e^2 (m+M)}} \qquad \dots \quad (1)$$

when the two clouds are of equal particle density $N_0$ and are moving in the opposite directions with the velocity $\vec{U}$.

There is no experimental evidence to support one view or the other. However, the recent satellite observations (Arnoldy *et al.* 1960) showing that the outer radiation belt disrupts during a magnetic storm and that the intensity of radiation in the outer belt is much higher (about five times the pre-storm value) near the end of the storm which stays at this value for about 8 to 10 days before it begins to decrease, seems to support indirectly the views developed in paper I (Tandon, unpublished).

Earlier authors did not consider the effect of gas pressure which in many situations (solar ion streams, laboratory experimentation) is of considerable significance. In this note we discuss the effect of the pressure of the two streams on the stability criteria. It can be easily seen from paper I that the protons do not appreciably change the stable length. We, therefore, assume that the protons provide a uniform back-ground of positive charge because of their heavier mass, although it may be mentioned that this assumption may not always hold good.

## COUNTERSTREAMING OF CLOUDS

Following paper I we suppose that a completely ionized neutral gas cloud of initial uniform density $N_{01}$ electrons per cm$^3$ is moving with initial uniform velocity $\vec{V}_{01}$. Let a similar stream with density $N_{02}$ electrons per cm$^3$ be moving with a velocity $-\vec{V}_{02}$. We shall further assume that the temperatures of the two gas clouds are the same and that the particle collisions are negligible.

After the interaction there will be perturbations in the densities, velocities and pressures. Assuming isothermal changes, let the perturbations be given by

$$N_1 = N_{01}+n_1 \qquad N_2 = N_{02}+n_2 \qquad \dots \quad (2)$$

$$\vec{V}_1 = \vec{V}_{01}+\vec{v}_1 \qquad \vec{V}_2 = -\vec{V}_{02}+\vec{v}_2 \qquad \dots \quad (3)$$

where the small quantities denote the perturbation values which are of the first order.

The basic equations are the equations of motion

$$\left[\frac{\partial}{\partial t}+(\vec{V}_1\,\text{grad})\right]\vec{V}_1 = \frac{e}{m}\vec{E}-\frac{1}{N_1 m}\,\text{grad}\,p_1 \qquad \dots \quad (4)$$

$$\left[\frac{\partial}{\partial t}+(\vec{V}_2\cdot\text{grad})\right]\vec{V}_2 = \frac{e}{m}\vec{E}-\frac{1}{N_2 m}\,\text{grad}\,p_2 \qquad \dots \quad (5)$$

and the equations of continuity

$$\left[\frac{\partial}{\partial t}+(\vec{V}_1\cdot\text{grad})\right]\vec{V}_1+N_1\,\text{div}\,\vec{V}_1 = 0 \qquad \dots \quad (6)$$

$$\left[\frac{\partial}{\partial t}+(\vec{V}_2\cdot\text{grad})\right]\vec{V}_2+N_2\,\text{div}\,\vec{V}_2 = 0 \qquad \dots \quad (7)$$

where $p_1$ and $p_2$ are the electron gas pressures of the two clouds respectively, which are given by

$$p_1 = N_1 kT \quad \text{and} \quad p_2 = N_2 kT \qquad \dots \quad (8)$$

and the electric field $\vec{E}$ is given by

$$\text{div}\,\vec{E} = 4\pi e(n_1+n_2) \qquad \dots \quad (9)$$

Combining equations (4) and (5) with the equations (2), (3), (6), (7) (8) and (9) we get

$$\left[\frac{\partial}{\partial t}+(\vec{V}_{01}\cdot\text{grad})\right]^2 n_1 = -\omega_1^2(n_1+n_2)-\frac{u_1^2}{N_{01}}\nabla^2 n_1 \qquad \ldots (10)$$

$$\left[\frac{\partial}{\partial t}-\vec{V}_{02}\cdot\text{grad})\right]^2 n_2 = -\omega_2^2(n_1+n_2)-\frac{u_2^2}{N_{02}}\nabla^2 n_2 \qquad \ldots (11)$$

where $\omega_1 = \left(\frac{4\pi e^2 N_{01}}{m}\right)^{\frac{1}{2}}$ and $\omega_2 = \left(\frac{4\pi e^2 N_{02}}{m}\right)^{\frac{1}{2}}$ are the two plasma frequencies and $u_1 = \left(\frac{N_{01}kT}{m}\right)^{\frac{1}{2}}$ and $u_2 = \left(\frac{N_{02}kT}{m}\right)^{\frac{1}{2}}$ are the Newtonian sound velocities.

Assuming the solutions of (10) and (11) of the form $e^{i\omega t+i\vec{k}\cdot\vec{r}}$, we have

$$\left\{[\omega+(\vec{V}_{01}\cdot\vec{k})]^2+\frac{u_1^2k^2}{N_{01}}-\omega_1^2\right\} n_{01} = \omega_1^2 n_{02} \qquad \ldots (12)$$

and

$$\left\{[\omega-(\vec{V}_{02}\cdot\vec{k})]^2+\frac{u_2^2k^2}{N_{02}}-\omega_2^2\right\} n_{02} = \omega_2^2 n_{01} \qquad \ldots (13)$$

Eliminating $n_{01}$ and $n_{02}$ from (12) and (13) we get the dispersion relation

$$\left[\omega+(\vec{V}_{01}\cdot\vec{k})\right]^2\left[\omega-(\vec{V}_{02}\cdot\vec{k})\right]^2-\frac{u_1^2k^2}{N_{01}}\left[w-(\vec{V}_{02}\cdot\vec{k})\right]^2+\frac{u_2^2k^2}{N_{02}}\left[w+(\vec{V}_{01}\cdot\vec{k})\right]^2$$

$$-\omega_2^2\left[\omega+(\vec{V}_{01}\cdot\vec{k})\right]^2-\omega_1^2\left[\omega-(\vec{V}_{02}\cdot\vec{k})\right]^2-k^2\left[\frac{u_1^2\omega_2^2}{N_{01}}+\frac{u_2^2\omega_1^2}{N_{02}}\right]+\frac{u_1^2u_2^2k^4}{N_{01}N_{02}}$$

$$= 0 \qquad \ldots (14)$$

we now put

$$\begin{aligned}\vec{V}_{01}+\vec{V}_{02} &= 2\vec{U}\\ \vec{V}_{01}-\vec{V}_{02} &= 2\vec{V}\\ p &= \omega+\vec{V}\cdot\vec{k}\\ \Omega &= \vec{U}\cdot\vec{k}\end{aligned} \qquad \ldots (15)$$

and get the simplified dispersion relation

$$(p^2-\Omega^2)^2-(\omega_1^2+\omega_2^2-\chi_1^2-\chi_1^2)(p^2+\Omega^2)+2p\Omega(\omega_1^2-\omega_2^2+\chi_2^2-\chi_1^2)$$
$$-(\chi_1^2\omega_2^2+\chi_2^2\omega_1^2)+\chi_1^2\chi_2^2 = 0 \qquad \ldots (16)$$

where $$\chi_1^2 = \frac{u_1^2k^2}{N_{e1}} \text{ and } \chi_2^2 = \frac{u_2^2k^2}{N_{e2}} \quad \ldots \quad (17)$$

Equation (16) is the general expression for the dispersion relation.

In order to get the physical insight, we assume that the two clouds have the same electron density. Thus $\omega_1^2 = \omega_2^2 = \omega_0^2$ (say) and $\chi_1^2 = \chi_2^2 = \chi^2$(say) and equation (16) reduces to

$$p^4 - 2p^2(\Omega^2 + w_0^2 - \chi^2) + \Omega^4 - 2\Omega^2(w_0^2 - \chi^2) - 2\chi^2 w_0^2 + \chi^4 = 0 \quad \ldots \quad (18)$$

which is quadratic in $p^2$.

For very low values of gas pressure, it can easily be seen that $\chi^2$ is negligibly small. Thus we have

$$p^4 - 2p^2(\Omega^2 + w_0^2) + \Omega^2(\Omega^2 - 2\omega_0^2) = 0 \quad \ldots \quad (19)$$

This is a similar expression as derived earlier in Paper I. From equation (18) it is evident that for all real values of $k$, $p$ is complex when

either $$\Omega^2 \leqslant 2\omega_0^2 - \chi^2 \quad \ldots \quad (20)$$

$$\Omega^2 > \frac{[illegible]}{4(\chi^2 - \omega_0^2)} \quad \ldots \quad (21)$$

showing thereby that the counterstreaming will be unstable by overstability. It appears, therefore, that the counterstreaming is always unstable. The fluctuations of protons will not change these conditions considerably. This, therefore, does not change basically any of the results deduced in Paper I.

ACKNOWLEDGMENT

One of us (JNT) is grateful to the University Grants Commission, India, for the award of Senior fellowship.

REFERENCES

Arnoldy, R. L., Hoffman, R. A. and Winckler, J. R., 1960, *J. Geophys Res.*, **65**, 1361.
Khan, F. D., 1957, *J. Fluid Mech.*, **2**, 601.
Parker, E. N., 1958, *Phy. Rev.*, **112**, 1429
Tandon, J. N., 1961, *Ind. J. Phy.*, **35**, 193.